NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3702

MEASUREMENTS OF ATMOSPHERIC TURBULENCE OVER A WIDE RANGE OF WAVELENGTH FOR ONE METEOROLOGICAL CONDITION

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SUMMARY

A method for obtaining a power spectrum of gust vertical velocity over a wide range of wavelength from simultaneous measurements made in flight has been devised and applied. This method has the advantage of not involving the use of airplane transfer functions. A gust vertical-velocity spectrum was determined for one meteorological condition for wavelengths from 10 feet to 60,000 feet. The test was conducted at low altitude in clear-air turbulence having a root-mean-square intensity of 5 feet per second. At the higher frequencies (shorter wavelengths), the power spectral density varied at a rate which was between the -5/3 power which has been predicted theoretically and the inverse square of the frequency. The spectrum which was obtained tended to flatten out for the longest test wavelengths. The break frequency which provides an indication of the scale of the turbulence occurred at a wavelength of approximately 6,000 feet. The results obtained indicate that the method of obtaining spectra used in this investigation is practicable.

INTRODUCTION

The application of the techniques of generalized harmonic analysis to the study of the effects of atmospheric turbulence on airplane design has created a need for detailed information on the power spectra of gusty air. The range of wavelength or frequency which is of interest in airplane design problems can be divided into three bands corresponding to the long-period (less than 0.1 cycle per second for a typical case), the short-period (0.1 to 1.0 cycle per second), and the very high frequency (1.0 to 30 cycles per second) airplane longitudinal response modes. Previous measurements of power spectra, such as those of references 1 to 3, have been restricted to one of the three bands, primarily because the measuring techniques which have been used were readily applicable to only one frequency band. Even though there have been enough measurements made over different ranges to cover the entire range of wavelengths of interest in airplane design problems, it is difficult to generalize over this wide range on the basis of the narrow-band measurements made under different conditions of turbulence scale and intensity.

The purpose of the present investigation has been to make a flight measurement of the vertical velocity component of atmospheric turbulence which covered all three frequency bands. The principal result was a power spectrum of the gust vertical velocity which covers a very wide range of wavelength for a single atmospheric condition. An additional result has been to demonstrate the feasibility of the technique for gathering a large amount of data for different turbulence conditions. This technique does not require the use of airplane transfer functions but involves instead the calculation of the incremental gust vertical velocity from the variations in airplane angle of attack, vertical velocity, and pitch attitude. A different method of calculating the gust vertical velocity was employed in each of the three frequency bands. The varied calculating techniques were used in the interest of expediency, since, in the low frequency and very high frequency bands, simplifying assumptions which greatly reduced the task of data reduction could be made.

SYMBOLS

α angle of attack, radians

α_o angle between longitudinal body axis and flight path, radians

γ flight-path angle, radians

θ pitch-attitude angle, radians

l distance from angle-of-attack vane to airplane center of gravity, ft

V airplane forward velocity, ft/sec

w vertical velocity, ft/sec

Subscripts:

a airplane

g gust

v vane

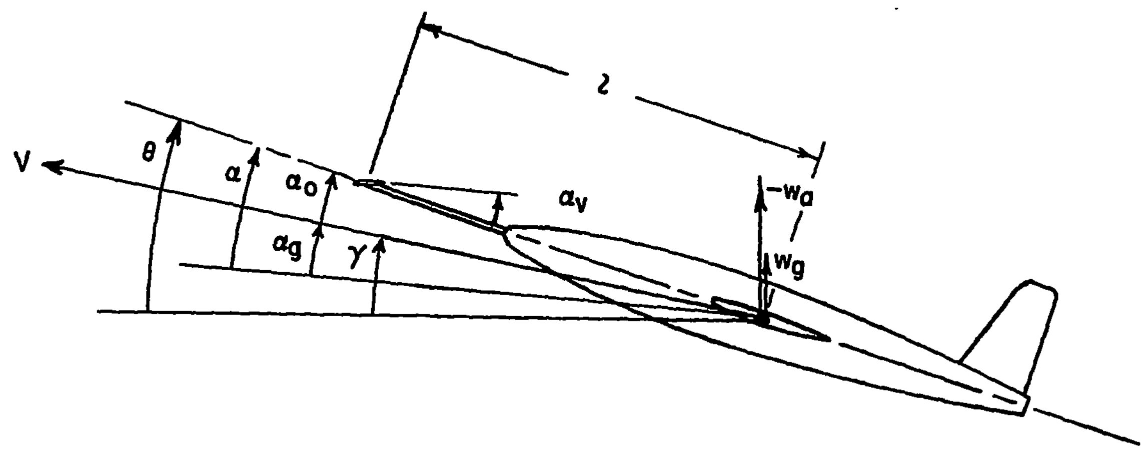
A dot over a symbol indicates the derivative with respect to time.

INSTRUMENTATION AND DATA REDUCTION

Measuring Technique

In order to facilitate the use of existing NACA recording instruments and to simplify the data-reduction procedure, the frequency range of interest was divided into three bands. The three frequency bands were defined as 0.01 to 0.1 cycle per second for the long-period band, 0.1 to 1.0 cycle per second for the short-period band, and 1.0 to 30 cycles per second for the very high frequency band.

The geometrical relationships for the airplane and flow-direction or angle-of-attack vane are shown in the following sketch:



All angles are assumed to be small variations from the steady-state condition, and small angle values are assumed for the trigonometric functions.

Based on the relations illustrated in the preceding sketch, the equation which relates gust vertical velocity to the airplane angle of attack, attitude, and vertical velocity is

$$w_g = V(\alpha - \theta) - w_a \tag{1}$$

This equation is derived from the knowledge that the angle of attack due to gust velocity is

$$\alpha_g = \alpha - \alpha_o$$

where the angle between the longitudinal body axis and the flight-path angle is

$$\alpha_o = \theta - \gamma$$

and

$$\gamma = -\frac{w_a}{V}$$

Equation (1) is modified for application in each of the three frequency bands as follows:

For the long-period band where $\Delta\alpha = 0$,

$$\Delta w_g \approx -V\ \Delta\theta - \Delta w_a \tag{2}$$

For the short-period band where $\alpha = \alpha_v + \frac{l\dot{\theta}}{V}$,

$$\Delta w_g = V(\Delta\alpha_v - \Delta\theta) - \Delta w_a + l\dot{\theta} \tag{3}$$

For the very high frequency band where $\alpha \approx \alpha_v$,

$$\Delta w_g \approx V\ \Delta\alpha_v \tag{4}$$

It should be noted that in the long-period band the angle of attack was considered to be constant, that in the short-period band the angle-of-attack indication of the vane was corrected for airplane pitching, and that in the very high frequency band the vane indication was used directly as a measure of gust velocity because the airplane pitching and vertical-velocity response could be considered to be negligible. Further discussion of the instrumentation and computation techniques which were used for each band of frequency is now presented.

Long-Period Measurements

Primary instrumentation for the long-period range consisted of a sun camera, a statoscope, and an airspeed-altimeter. The sun camera measured pitch attitude of the airplane and the statoscope measured variations in pressure altitude which were differentiated to obtain vertical-velocity variations of the airplane; both measurements were required for use in equation (2). The sun camera, of course, has a disadvantage in that it requires clear skies and a flight path toward the sun. However, the sun camera was used rather than a gyroscopic attitude recorder because it was not certain that a sufficiently precise gyroscopic instrument was readily available. The sun camera allowed measurement of attitude to within about $\pm 0.15^{\circ}$ which corresponds to an error in vertical velocity of 3/4 foot per second. In the worst case for the statoscope at a frequency of 0.10 cycle per second, the smallest change in vertical velocity which could be detected was about $1\frac{1}{2}$ feet per second. The resultant maximum error in gust velocity was therefore about 2 feet per second. Supplementary instrumentation for the long-period band consisted of three control-position recorders.

In order to provide for an extra-long record, a film speed of 1/8 inch per second was used with a timer with a one-second interval. The long-period sample was 51 minutes long and corresponded to approximately 170 air miles at the mean airspeed of 290 feet per second. The root-mean-square or standard deviation of the airspeed was approximately 8 feet per second, and in the data reduction the airspeed was considered to be constant. The data were read at intervals of 5 seconds. The power spectrum was computed by digital methods for the frequency range from 0.005 to 0.1 cycle per second. This frequency range corresponds to the band of gust wavelengths between approximately 60,000 feet and 3,000 feet. The band width for independent estimates of power was 0.005 cycle per second.

Short-Period Measurements

Instrumentation for the short-period range included a sensitive normal accelerometer, angle-of-attack vane, and a pitch-angular-velocity recorder. Supplementary instruments consisted of a three-component accelerometer and roll- and yaw-angular-velocity recorders. The angle-of-attack vane was mounted 44 inches ahead of the nose of the airplane on a boom (fig. 1). Indications from the angle-of-attack vane were corrected for induced velocity at the vane due to rate of pitching. The error in angle of attack due to upwash at the vane was estimated to be approximately 8 percent and could properly be neglected in the determination of gust spectra.

The pitch-rate record was also integrated to obtain airplane pitch-attitude variations in this frequency range. Airplane vertical velocity was obtained by integrating the normal-acceleration record. The maximum error in gust velocity was about 5 feet per second at 0.05 cycle per second and decreased to 1/4 foot per second at 1 cycle per second. Film speed was 1/2 inch per second with a 1/2-second timer.

The two 9-minute short-period samples were read at 1/2-second intervals and the power spectrum of gust vertical velocity was obtained in the same manner as for the long-period range. The spectrum was determined over the range of frequencies 0.05 to 1.0 cycle per second corresponding to the range of gust wavelengths between 6,000 feet and 300 feet. The band width for independent estimates of power was 0.05 cycle per second.

Very High Frequency Measurements

For measuring gust vertical velocity in the very high frequency range, a mass-balanced balsa angle-of-attack vane was employed. This vane had a natural frequency of 50 cycles per second and had approximately 60-percent critical damping at the test speed and altitude. This vane was located on a boom approximately 50 inches from the nose of the airplane. The angle of attack indicated by the vane was used directly to compute gust vertical velocity as indicated in equation (4). No corrections for upwash or induced velocity due to pitch rate were included. Except in the region of boom resonance, the measure of gust velocity should be within ±1/4 foot per second. It should be noted that at or near the maximum frequency included by this investigation a possible error of 1/4 foot per second will be more significant than the larger possible error noted for the lower frequency bands because of the greatly reduced gust velocities encountered in the very high frequency range.

In order to provide a high degree of frequency resolution, a film speed of 4 inches per second with a timer with a 1/10-second interval was used. The two 60-second very high frequency samples were read at 1/60-second intervals and the power spectrum of gust vertical velocity was obtained in the same manner as in the preceding cases. The spectrum was determined over the range of frequencies from 0.5 to 30 cycles per second corresponding to the range of gust wavelengths between 600 feet and 10 feet. The band width for independent estimates of power was 1.0 cycle per second.

TESTS

Measurements of atmospheric turbulence were made simultaneously for the three frequency bands during a test run of 170 air miles from the upper end of the Chesapeake Bay to a point off Norfolk, Va. A jet fighter airplane was used for the test vehicle. The approximate track of the flight and the location of the test samples are shown in figure 2. Data for the long-period range were recorded for the whole distance. The sample marked 1 in figure 2 was 51 minutes in length. Nine-minute samples for the short-period frequency band marked 2 and 5 in the figure were obtained near the beginning and near the end of the flight. The two 1-minute very high frequency samples marked 3 and 4 were recorded during the first short-period sample. The programing of the samples was designed to provide an indication of the consistency of the turbulence.

The measurements were made in clear, moderately turbulent air which was turbulent primarily at low altitude and which was the byproduct of the northeasterly offshore passage of a storm center. There was a 25-knot northwest wind throughout the test runs. All the samples were obtained at a pressure altitude of 1,700 feet at a mean airspeed of

290 feet per second. The test run was started at 12m e. s. t. on January 20, 1955. The methods used herein were designed so that control-fixed conditions were not required. It was possible, therefore, for the pilot to make normal corrections in order to hold speed and altitude within a specified range.

RESULTS AND DISCUSSION

The results of the measurements of gust vertical velocity are presented in the form of power spectral densities for the separate frequency bands in figures 3, 4, and 5 and the corresponding probability distributions are shown in figures 6, 7, and 8. The composite wide-band spectrum is presented in figure 9.

Figure 3 shows the gust vertical-velocity spectrum for the long-period measurements. A 95-percent confidence band, based on the statistical reliability of a sample as set forth in reference 4 is shown for an arbitrary smooth fairing of the spectrum. For values of inverse wave-length less than 0.000017 cycle per foot, the confidence band widens rapidly. Figure 4 presents the two short-period spectra with a 95-percent confidence band based on an arbitrary, straight-line fairing of mean values. These two samples which were obtained about 130 miles apart have approximately the same spectral shape and intensity. The very high frequency samples of figure 5 were obtained about 20 miles apart and, again, agree fairly well in shape and intensity. The 95-percent confidence limits are again shown for an arbitrary, straight-line fairing of mean values. In both the short-period samples of figure 4 and the very high frequency samples of figure 5, some data were obtained which overlap the adjacent lower frequency band. The sharp peak in power at 0.024 cycle per foot (7 cycles per second) shown in figure 5 is the result of resonance of the boom on which the angle-of-attack vane was mounted and should be disregarded.

The probability distributions of figures 6, 7, and 8 are presented for comparison with the normal or Gaussian distribution. For some purposes the spectrum provides sufficient information to describe the effect of gusts on a particular system. For example, the response spectrum and standard deviation can be determined. However, many applications, such as computation of time on target or number of zero crossings, which can be obtained from spectral information depend upon the assumption that the process has a normal distribution. Each of the probability distributions were obtained from the gust time history computed for the corresponding frequency band and therefore represent gust velocities which have been passed through a band-pass filter. The probability distribution of figures 6, 7, and 8 appear to be reasonably normal.

Figure 9 shows the composite spectrum over the whole range of frequencies for which data were obtained. In terms of wavelengths, this range was from 10 feet to 60,000 feet. The spectra for the two higher frequency bands are in each case the average of spectra from two samples. As a result, the 95-percent confidence bands are narrower for these two frequency bands than was the case in figures 4 and 5. The root-mean-square gust velocity for the whole 51-minute test interval was slightly greater than 5 feet per second. According to the data of reference 5, gust intensities of this magnitude should be encountered not more than 20 percent of the time in clear-air turbulence or not more than 6 percent of the total flight time at low altitude. The intensity difference between the short-period spectrum and the very high frequency spectrum in the overlapping region is largely a result of folding higher frequency power into the short-period spectrum. This is an effect resulting from the analysis of sampled data (ref. 4). The same result has been obtained from several subsequent measurements of power spectra for two overlapping frequency bands. The effect is also present in the overlap between the long- and short-period spectra but the result is not so pronounced. The root-mean-square gust intensity was calculated for each of the nine 1 minute intervals of the first short-period record. The variation from the mean was no more than ±20 percent. The composite spectrum is therefore probably best represented by a smooth fairing of the data of figure 9.

At the higher frequencies the variation of power spectral density was at a rate between the -5/3 power predicted by the theory of reference 6 and the inverse square of the frequency. This rate of intensity variation is in substantial agreement with the experimental results of references 2 and 3. The spectrum tended to flatten out at the lowest test frequencies or longest wavelengths. The band of wavelengths covered by the measurements was broad enough to show that the asymptotic break in the spectrum occurs at a wavelength of about 6,000 feet and thus provides an indication that the scale of the turbulence as commonly defined (as in ref. 5, for example) was about 1,000 feet.

It should be noted that the procedure used in this investigation which involved film records and digital computation methods is much more laborious than will be the case when airborne tape recorders become available for use with existing harmonic analyzers for the data reduction.

CONCLUDING REMARKS

The results of this investigation show that it is practicable to obtain a power spectrum of gust vertical velocity for a wide range of frequencies from a set of simultaneous measurements made in flight. Such a spectrum was obtained for one meteorological condition for a range of wavelengths of from 10 feet to 60,000 feet. At the higher frequencies (shorter wavelengths) the rate of variation of intensity was between the -5/3 power predicted by theory and the inverse square of the frequency. The spectrum tended to flatten out for the longest test wavelengths. The spectrum break occurred at a wavelength of about 6,000 feet. It should be noted that the procedure used in this investigation which involved film records and digital computation methods is much more laborious than will be the case when airborne tape recorders become available for use with existing analog harmonic analyzers for the data reduction.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., March 12, 1956.

REFERENCES

1. Clementson, Gerhardt C.: An Investigation of the Power Spectral Density of Atmospheric Turbulence. Ph. D. Thesis, M.I.T., 1950.

2. Chilton, Robert G.: Some Measurements of Atmospheric Turbulence Obtained From Flow-Direction Vanes Mounted on an Airplane. NACA TN 3313, 1954.

3. Notess, Charles B., and Eakin, Grady J.: Flight Test Investigation of Turbulence Spectra at Low Altitude Using a Direct Method for Measuring Gust Velocities. Rep. No. VC-839-F-1 (Contract AF 33(616)174), Cornell Aero. Lab., Inc., July 1, 1954.

4. Tukey, John W.: The Sampling Theory of Power Spectrum Estimates. Symposium on Applications of Autocorrelation Analyses to Physical Problems (Woods Hole, Mass.), June 13-14, 1949, pp. 47-67. (Sponsored by ONR, Dept. Navy.)

5. Press, Harry, and Meadows, May T.: A Reevaluation of Gust-Load Statistics for Applications in Spectral Calculations. NACA TN 3540, 1955.

6. Heisenberg, W.: Zur statistischen Theorie der Turbulenz. Zeitschrift für Physik, vol. 124, 1948, pp. 628-657.

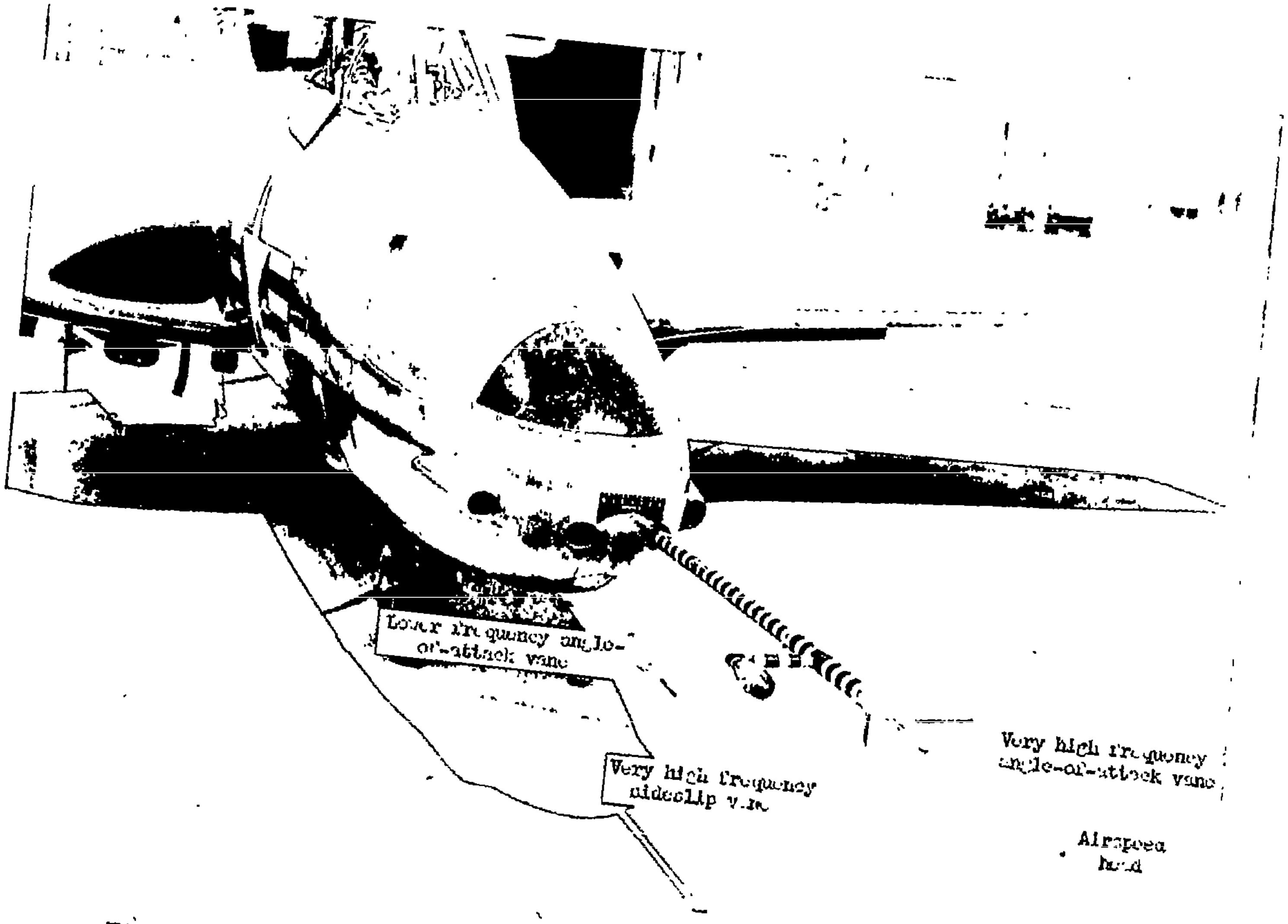


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Figure 1.- Photograph of nose-boom installation of flow-direction vanes and airspeed head.

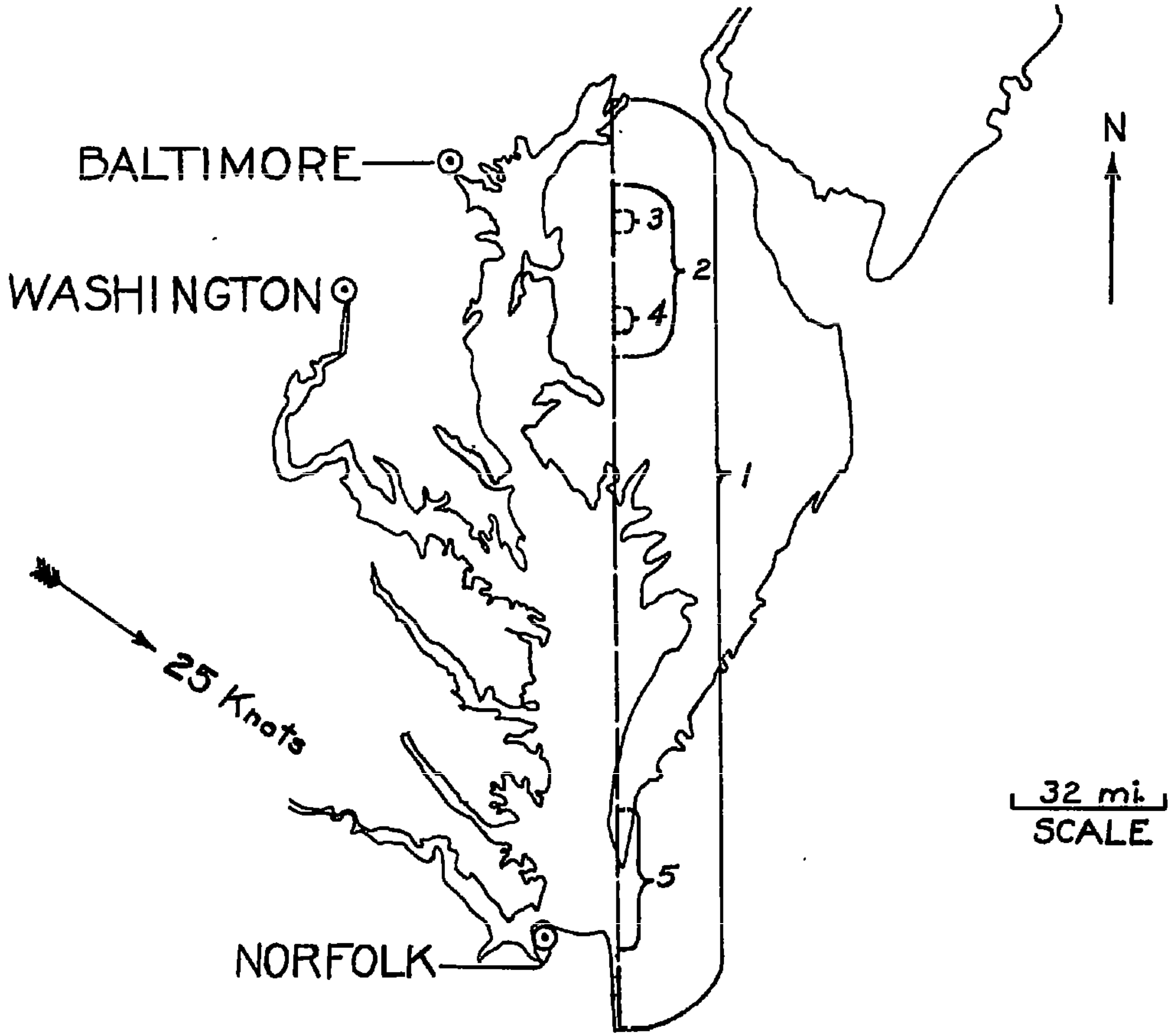

Figure 2.- Track of gust-measurement flight with location of test samples.

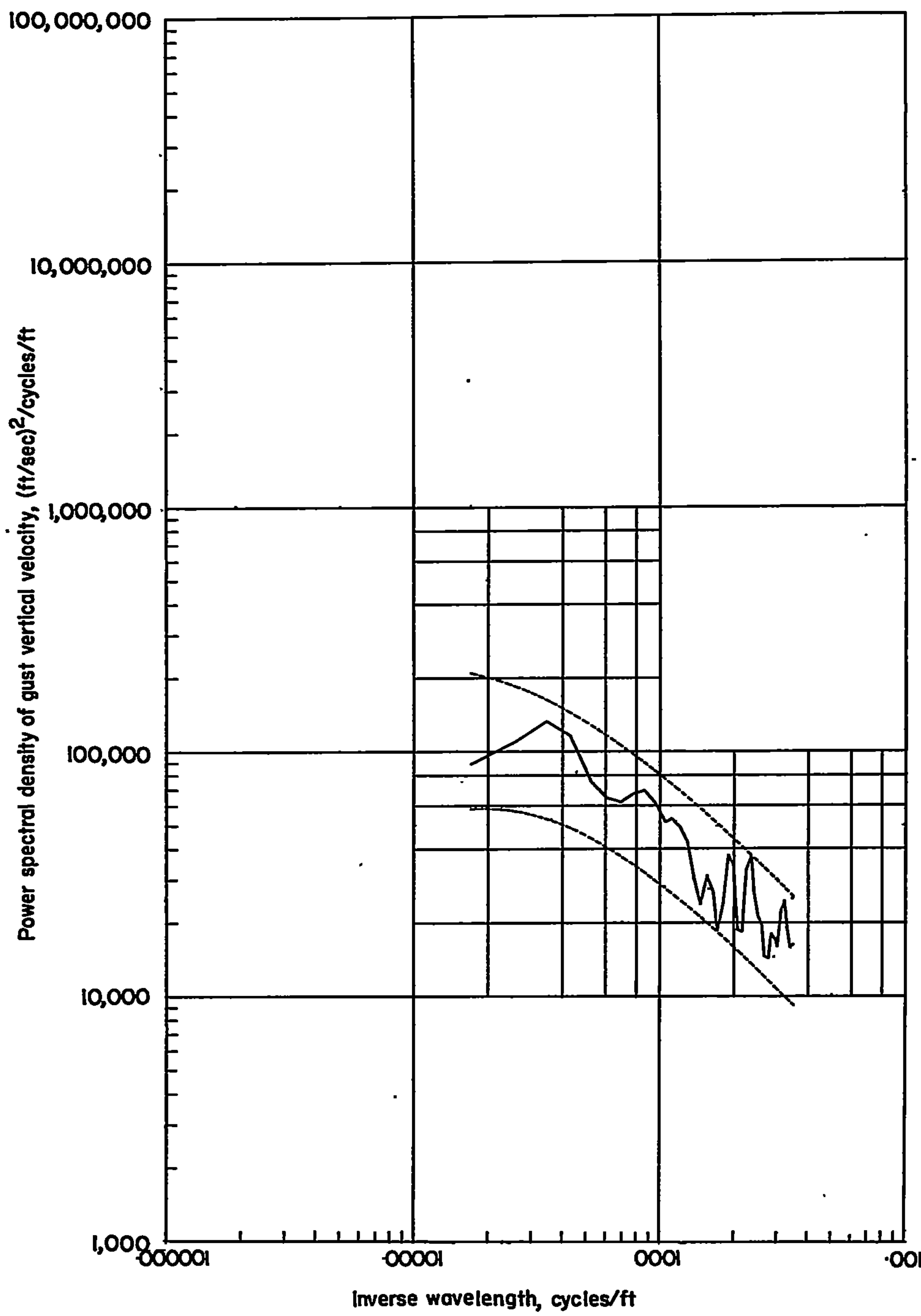

Figure 3.- Spectrum of gust vertical velocity for the long-period band for wavelengths from 5,000 feet to 60,000 feet. A 95-percent confidence band is shown for an arbitrary smooth fairing.

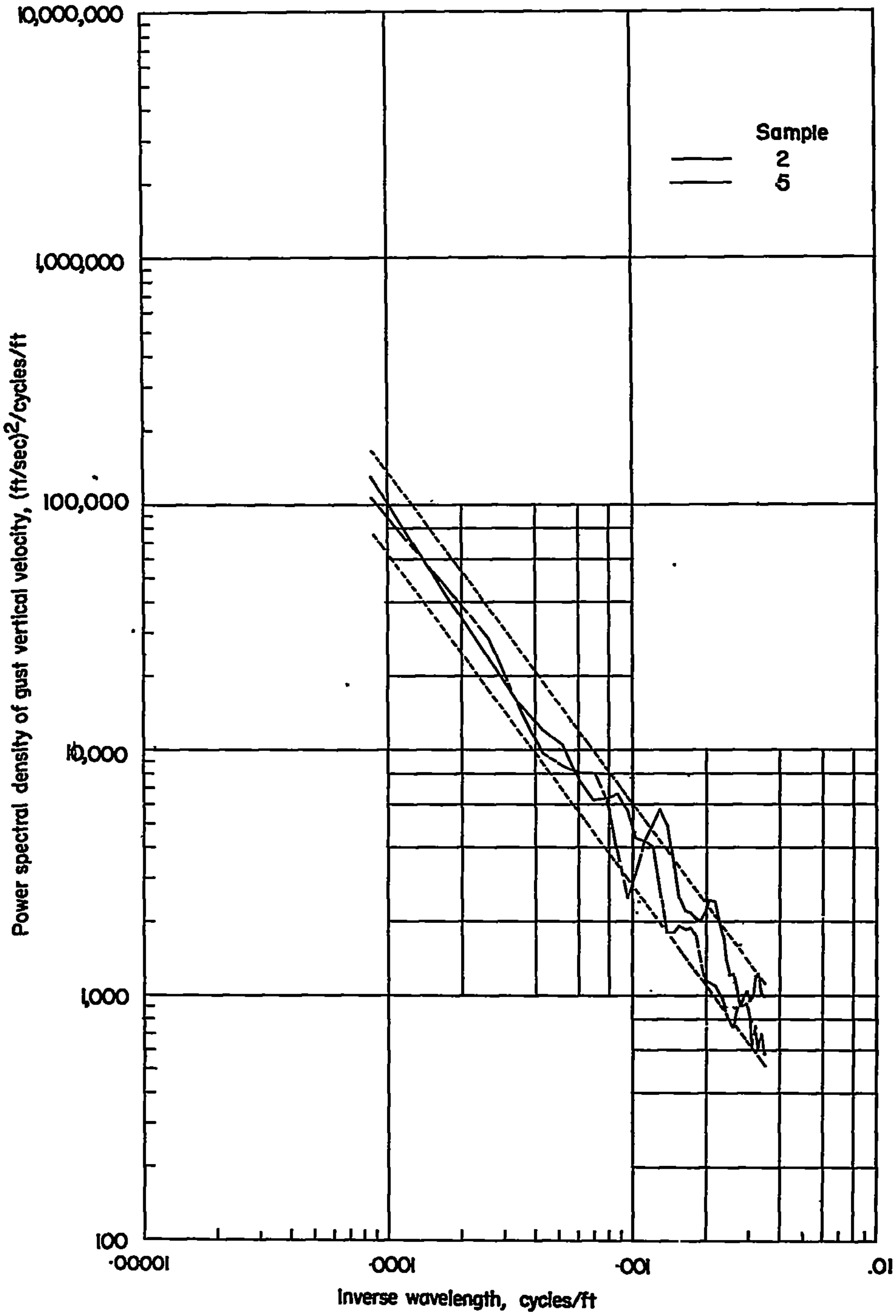

Figure 4.- Spectra samples of gust vertical velocity for the short-period band for wavelengths from 300 to 6,000 feet. A 95-percent confidence band is shown for an arbitrary smooth fairing.

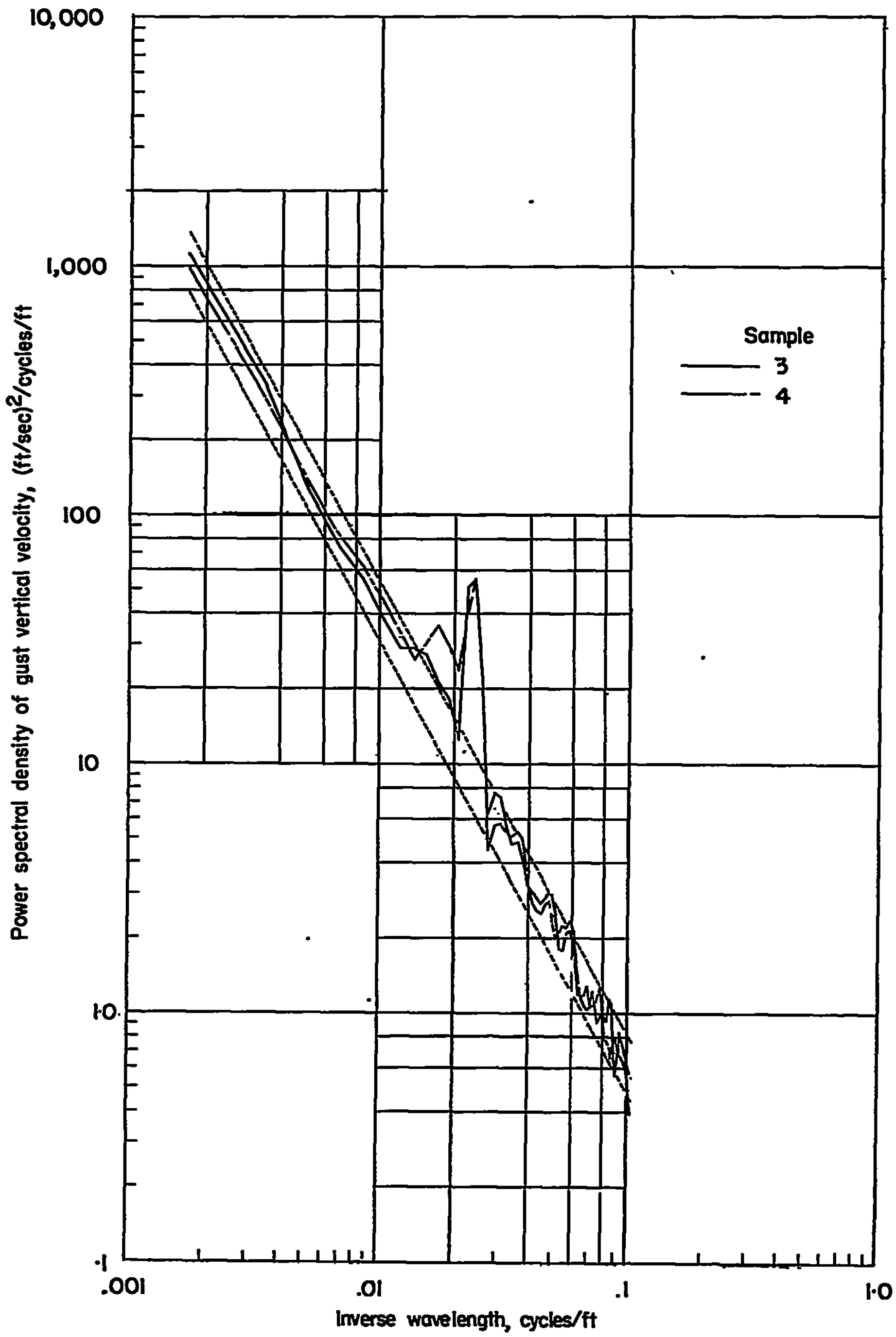

Figure 5.- Spectra samples of gust vertical velocity for the very high frequency band for wavelengths from 10 feet to 300 feet. A 95-percent confidence band is shown for an arbitrary straight-line fairing.

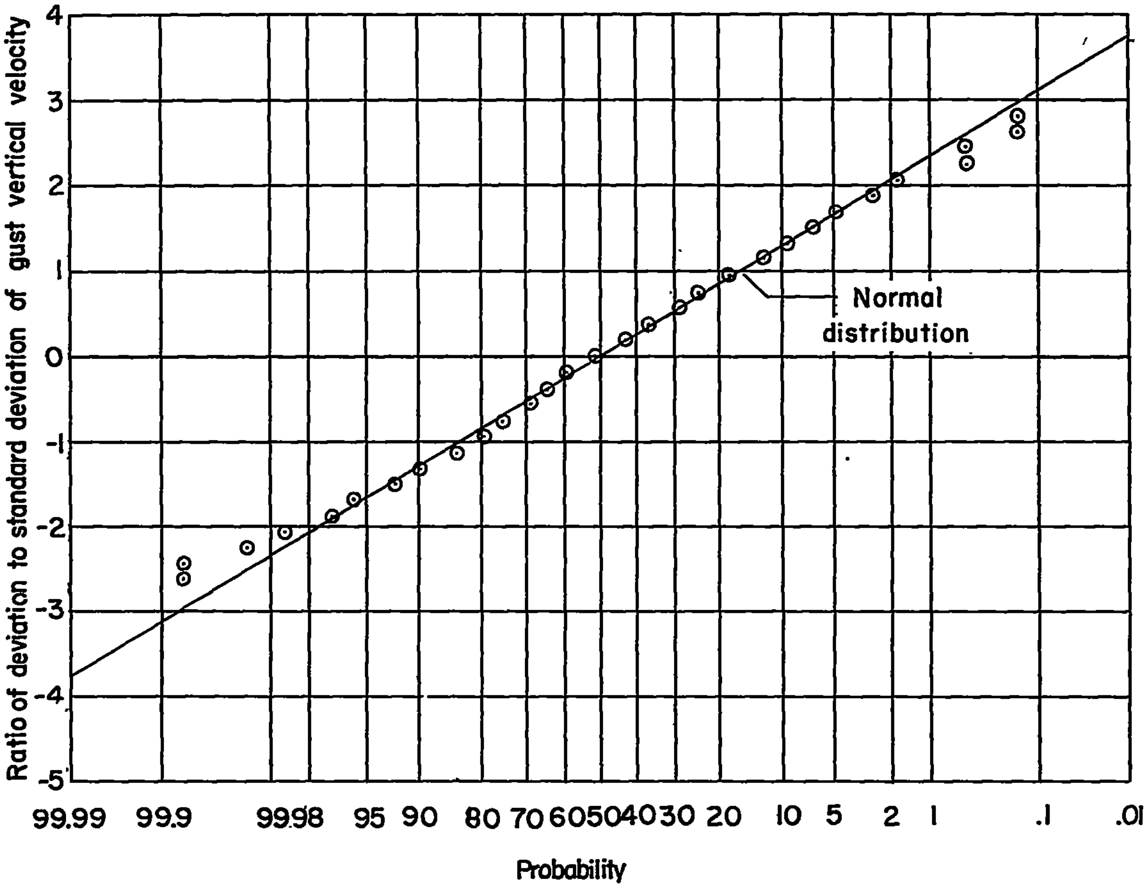

Figure 6.- Probability that ratio of deviation to standard deviation of gust vertical velocity will exceed a given value as obtained from 51-minute long-period sample.

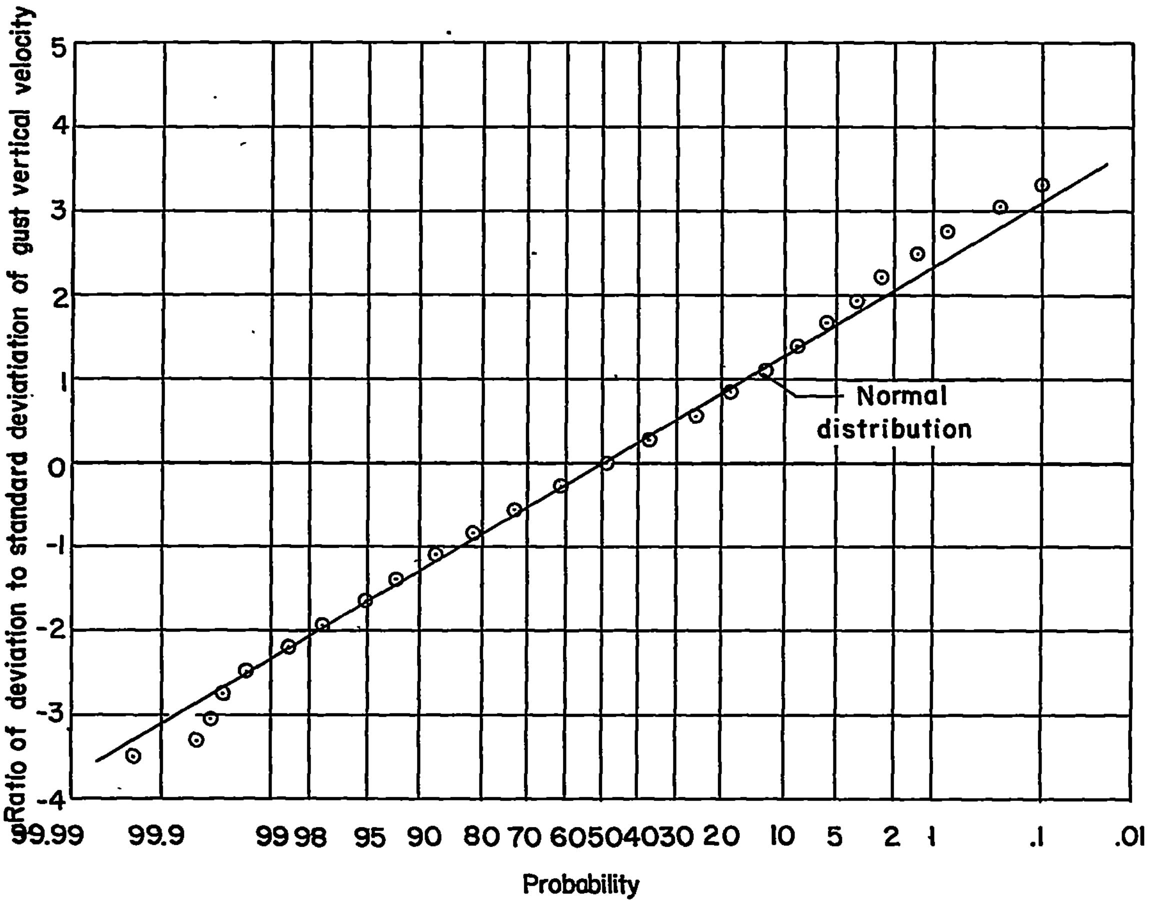

Figure 7.- Probability that ratio of deviation to standard deviation of gust vertical velocity will exceed a given value as obtained from the two 9-minute short-period samples.

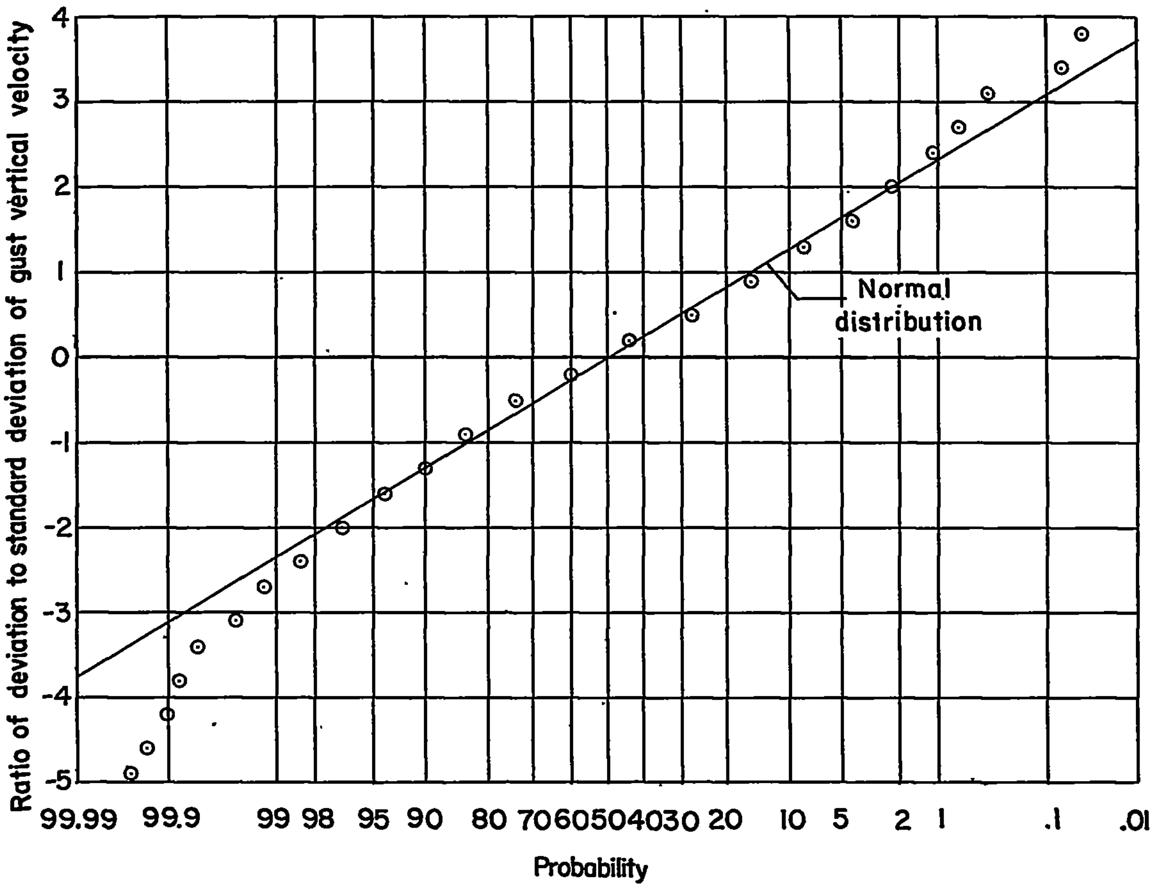

Figure 8.- Probability that ratio of deviation to standard deviation of gust vertical velocity will exceed a given value as obtained from the two 60-second very high frequency samples.

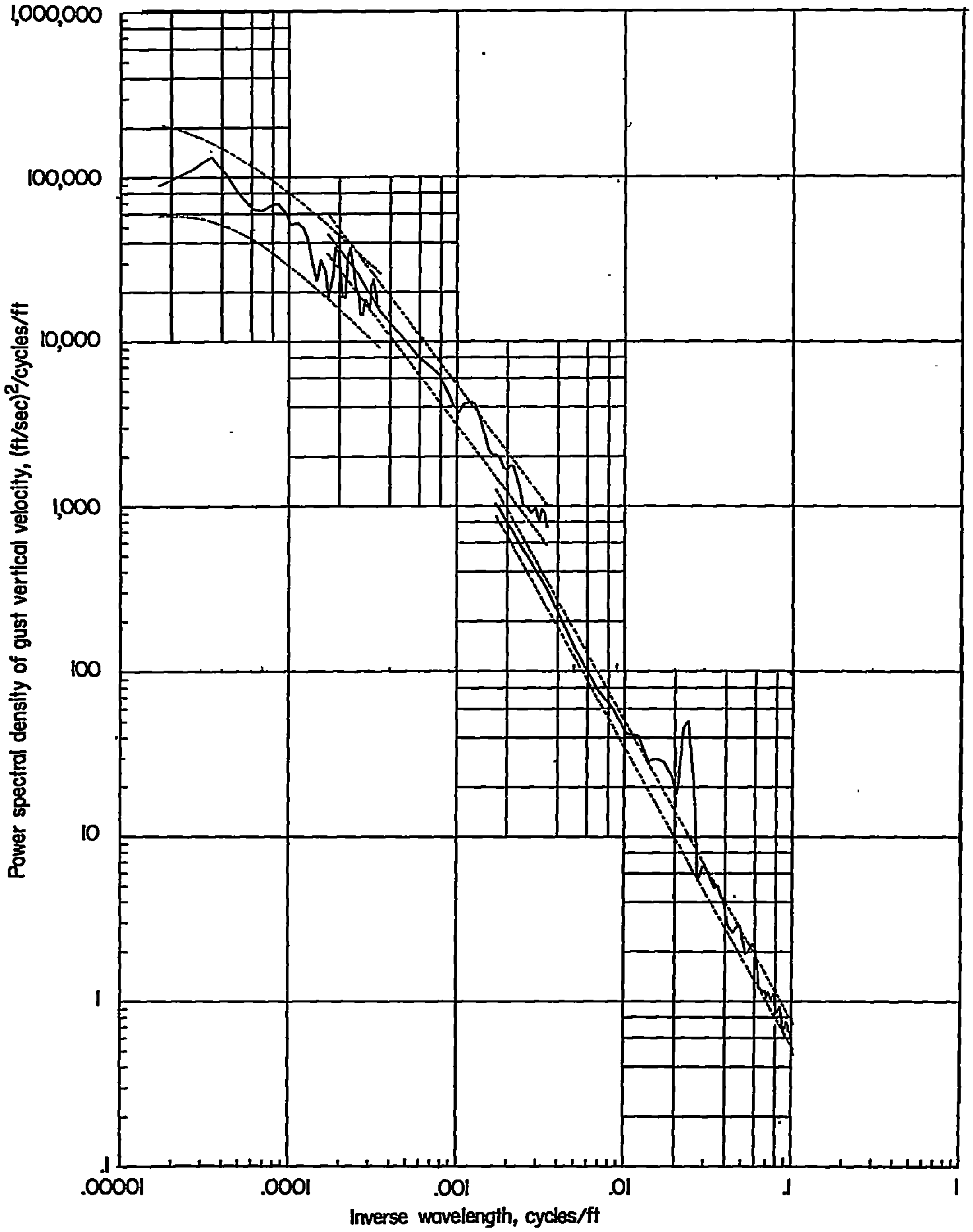

Figure 9.- Composite wide-band spectrum of gust vertical velocity for wavelengths from 10 feet to 60,000 feet. 95-percent confidence bands are shown for arbitrary smooth fairings.